On Complexity and Emergence

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February 3, 2008

Abstract

Numerous definitions for *complexity* have been proposed over the last half century, with little consensus achieved on how to use the term. A definition of complexity is supplied here that is closely related to the Kolmogorov Complexity and Shannon Entropy measures widely used as complexity measures, yet addresses a number of concerns raised against these measures. However, the price of doing this is to introduce context dependence into the definition of complexity. It is argued that such context dependence is an inherent property of complexity, and related concepts such as entropy and emergence. Scientists are uncomfortable with such context dependence, which smacks of subjectivity, and this is perhaps the reason why little agreement has been found on the meaning of these terms.

1 The problem of Complexity

In the last 15 years, the study of *Complex Systems* has emerged as a recognised field in its own right, although a good definition of what a complex system actually is has eluded formulation. Attempts to formalise the concept of *complexity* go back even further, to Shannon's inception of *Information Theory*[14]. A good survey of the tortuous path the study of complexity has followed is provided in Edmonds[4]. Of particular importance to this paper is the concept of *Kolmogorov Complexity* (also known as *Algorithmic Information Complexity*) introduced independently by Kolmogorov[10], Chaitin[2] and Solomonoff[15]. Given a particular universal Turing Machine[1] (UTM) U, the Kolmogorov complexity of a string of characters (ie a description) is the length of the shortest program running on U that generates the description.

There are two main problems with Kolmogorov complexity:

[1] A Turing Machine is a formal model of a digital computer

1. The dependence on U, as there is no unique way of specifying this. Even though the Invariance theorem[11, Thm 2.1.1] guarantees that any two UTMs U and V will agree on the complexity of a string x up to a constant independent of x, for any descriptions x and y, there will be two machines U and V disagreeing on whether x is more complex that y, or vice-versa.

2. Random sequences have maximum complexity, as by definition a random sequence can have no generating algorithm shorter than simply listing the sequence. As Gell-Mann[8] points out, this contradicts the notion that random sequences should contain no information.

The first problem of what reference machine to choose is a symptom of context dependence of complexity. Given a description x, any value of complexity can be chosen for it by choosing an appropriate reference machine. It would seem that complexity is in the eye of the beholder[5]. Is complexity completely subjective? Is everything lost?

Rather than trying to hide this context dependence, I would prefer to make it a feature. Instead of asserting complexity is a property of some system, it is a property of descriptions (which may or may not be about a system). There must also be an interpreter of these descriptions that can answer the question of whether two descriptions are equivalent or not. Consider Shakespeare's *Romeo and Juliet*. In Act II,ii, line 58, Juliet says "My ears have yet not drunk a hundred words". If we change the word "drunk" to "heard", your average theatre goer will not spot the difference. Perhaps the only one to notice would be a professional actor who has played the scene many times. Therefore the different texts differing by the single word "drunk/heard" in this scene are considered equivalent by our hypothetical theatre goer. There will be a whole *equivalence class* of texts that would be considered to be *Romeo and Juliet* by our theatre goer.

Once the set of all possible descriptions are given (strings of letters on a page, base pairs in a genome or bits on a computer harddisk for example), and an equivalence class between descriptions given, then one can apply the Shannon entropy formula to determine the complexity of that description, under that interpretation:

$$C(x) = \lim_{\ell\to\infty} \ell \log_2 N - \log_2 \omega(\ell, x) \tag{1}$$

where $C(x)$ is the complexity (measured in bits), $\ell(x)$ the length of the description, N the size of the alphabet used to encode the description and $\omega(\ell, x)$ the size of the class of all descriptions of length less than ℓ equivalent to x. We assume that the interpreter of a description is able to determine where a description finishes, so that a description y of length $\ell(y)$ is equivalent to all $N^{\ell-\ell(y)}$ length ℓ descriptions having y as a prefix.

If we choose our description set to be all bitstrings, and our equivalence class to be all bitstrings that produce the same output when executed by a universal

Turing machine U, then

$$\omega(\ell, x) = \sum_{p:U(p)=U(x),\ell(p)\leq\ell} 2^{\ell-\ell(p)}, \tag{2}$$

where $\ell(p)$ is the length of program p. As $\ell \to \infty$, this distribution (when normalised) is known as the *universal a priori probability*[11, Def 4.3.3]. By the *coding theorem*, the complexity defined by equation (1) is equal to the Kolmogorov complexity up to a constant independent of x.

This perspective helps us understand the problem of random strings having maximal complexity. In an equivalence class generated by a human observer, one random string is pretty much the same as any other. Therefore the ω term of a completely random string will large, probably of comparable size to N^ℓ. Therefore the complexity of a random string, as interpreted by a human observer is low, exactly the property required of Gell-Mann's *effective complexity*.

Whilst context dependence would appear to open up the curse of subjectivity, it needn't necessarily do so. In many situations, the equivalence relation is well defined. For example the notion of species in biology is reasonably well defined (although disagreement exists in a number of cases). This could, in principle, along with a detailed knowledge of the genetic code, be used to estimate the complexity of different species. This principle has been used in a number of artificial life studies[1, 16] for studying the evolution of complexity.

2 Emergence

Emergence is that other area of complex systems study that has experienced controversy and confusion. Its importance stems from the belief that emergence is the key ingredient that makes complex systems complex. Putting things colloquially, emergence is the concept of some new phenomenon arising in a system that wasn't in the system's specification to start with. There is some considerable debate as to how this happens, or whether emergence can truly happen within a formal system such as an agent-based model[13, 7].

Let me illustrate this debate with the example of gliders in *The Game of Life*. I would contend that this phenomenon is emergent, in the sense that the glider concept is not contained within the cellular automaton implementation language — namely states and neigbourhoods. This puts me at odds with Rosen, who would argue that gliders are but complicated combinations of simple machines (the cellular automata), not examples of *complexity*[2]

What is my definition of emergence then? To set the scene, let me introduce two descriptions of a system, called the *microdescription* and *macrodescription*, each coded in their own language. Ronald et al., use the term $\mathcal{L}_1$ and $\mathcal{L}_2$ to refer

[2]Rosen uses the word complexity as a quality, rather like emergence is used in this paper, as opposed to a quantity.

to the micro- and macrolanguage respectively[12]. They call the microlanguage the "language of design" in view of artificial life applications. However, the microdescription may equally well be our best description of what happens at the most fundamental level. To make it clear, emergence is not due to the failure of the microdescription as a modeling effort, since the emergent property should still appear as the result of a computer simulation constructed using the microdescription.

We also assume that the macrodescription is a *good theory*. There is, in general, a trade-off between the predictive power of a theory, and its explanatory power. Of course, a theory may be neither predictive nor explanatory, but in this case the theory is not so good, and would be rejected in favour of one that is better. One can, of course, produce examples of emergent concepts based on a poor macrodescription, but in this case the correspondence with the real or simulated system would be lacking, and the concept would not be an observed phenomenon.

An emergent phenomenon is simply one that is described by atomic concepts available in the macrolanguage, but cannot be so described in the microlanguage. In the case of the glider in The Game of Life, any attempt at describing a glider would involve the CA transition table (naturally), but also the specific pattern of cell states that make up the glider. But which pattern? A glider can appear at any location within the CA, and may have one of four possible orientations. The description cannot represent the fact that two gliders separated diagonally by 1 cell in along each axis with the same orientation are temporally related. A glider, as an object-in-itself, is a pure macrodescription object.

Ronald et al. focus on the element of *surprise* as a test of emergence. In this, they are trying to capture emergence as some kind of dissonance between the micro and macro languages. To be fair to their work, they claim only a test for emergence, not a definition, along the lines of the more famous Turing test. However, the surprise factor really only works when the macrolanguage is enlarged (by the emergent concept) in order to make the macrodescription a better model of the system. Once the emergent property has been recognised by the observer, the property is no longer surprising. The definition of emergence given here works, regardless of whether the observer is still surprised or not.

Of considerable interest is, given a system specified in its microlanguage, does it have emergent properties? There is no general procedure for answering this question. One has to construct a macrodescription of the system. If this macrodescription contains atomic concepts that are not simple compounds of microconcepts, then one has emergent properties. Is there a best macrodescription for any given system? This question is outside of the scope of this paper, and needs to be answered by theories of how scientific theories are developed. However, in general, it seems unlikely that there would be a "best description", as it depends on the motives of the person using the description. For example, I have already alluded to the tension between predictive power and explanatory power.

3 Entropy as a case study of emergence

Thermodynamics provides an excellent case study in emergence, as it is well understood theory, and also illustrates the link between emergence and complexity.

Thermodynamics is a macroscopic description of material systems, expressed in terms of concepts such as temperature, pressure and entropy. It is related to the microscopic description of molecular dynamics via the reductionist theory of *statistical mechanics*.

Within thermodynamics, entropy is defined by differences along a reversible path:

$$\Delta S = \Delta Q/T$$

Within the framework of thermodynamics, this quantity is objectively defined, up to an additive constant (usually assumed to be such that entropy vanishes at absolute zero).

Within Statistical Mechanics, entropy in the microcanonical ensemble is given by the Boltzmann formula:

$$S = k_B \ln W, \tag{3}$$

where k_B is the Boltzmann constant (giving entropy in units of Joules per Kelvin), and W is the number of microstates accessible to the system for a particular macrostate. This formula is very similar to the information-based complexity formula (1), and this led Jaynes[9] to remark that entropy "measures our degree of ignorance as to the unknown microstate". Denbigh and Denbigh[3] are at pains to point out that entropy is fully objective, as it only depends on the macroscopic quantities chosen to define the macroscopic state (the temperature, pressure and so on), and not dependent on any extraneous observer related property. Provided two observers choose the same microscopic and macroscopic description of the system, they will agree on the value of entropy. By my previous definition of emergence, entropy is an emergent quality of the system. This example illustrates how context dependent emergent properties can be fully objective.

With entropy defined by (3), the well known H-theorem holds. As a consequence, the macroscopic (thermodynamic) description is time irreversible, whereas the microscopic description is reversible. Time irreversibility is likewise an emergent property of this system.

The very clear relation between the Boltzmann-Gibbs entropy (3), and complexity (1) indicates that complexity is itself an emergent concept. If the microscopic language and macroscopic language were identical, corresponding to a situation of no emergence, complexity of descriptions degenerates to the trivial measure of description length.

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